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Are Anomalously Short Tunnelling Times Measurable?

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Low and Mende have analyzed the conditions that would make possible an actual measurement of an anomalously short traversal time through a potential barrier concluding that such a measurement cannot be made because it is not possible to describe the tunnelling of a wave packet initially close to the barrier by the "usual wave packet space time analysis". We complement this work in several ways: It is argued that the described failure of the usual formalism occurs under a set of too restrictive conditions, some of them not physically motivated, so it does not necessarily imply the impossibility of such a measurement. However, by retaining only conditions well motivated on physical grounds we have performed a systematic numerical check which shows that the conclusion by Low and Mende is indeed generally valid. It is shown that, as speculated by Low and Mende, the process is dominated by over the barrier transmission. © 1996 Academic Press, Inc.

1. INTRODUCTION

Determining the "duration of a quantum collision" is a nontrivial matter, especially when different outgoing channels are involved, i.e., when we try to answer the question: "What is the duration of a collision for the particles emerging in a particular outgoing channel?". The difficulties can be traced back to the fact that in general the projection operators selecting the part of the wave in the interaction region, D, and the part of the wave that will come out in the selected outgoing chanel, P, do not commute [1]. We are thus faced with the old problem of quantizing a pair of quantities associated with non-commuting operators. A universal receipe to solve this problem does not exist. The so called "tunnelling time problem" represents a particularly important realization of this general question. In its simplest form, the outgoing channels correspond to reflection and transmission of a structureless particle through a one dimensional barrier. In this case the question turns out to be: "How long does it take a particle to cross the barrier?" Quantum tunnelling complicates things, since the more quantum a collision becomes the more severe will be the effects of the non-commutativity of P (associated in this case with transmission) and D.

The ellusive tunnelling times have been challenging theoretitians and experimentalists alike for a long time. Recent interest in the subject, triggered by a seminal paper by M. Buttiker and R. Landauer in 1982 [2], has been motivated in part by the possible applications of tunnelling in semiconductor technology, and by the

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search for the appropriate time scale that determines whether tunnelling is "sudden" or "adiabatic" with respect to the characteristic times of the additional degrees of freedom coupled to the tunnelling traslational motion.

There are several reviews on the subject discussing and comparing different answers [3–7]. We shall concentrate here on one particularly intriguing aspect of the problem: The superluminal velocities or anomalously short traversal times implied by some of the approaches. Such paradoxical results are found for example in the so called "extrapolated stationary phase times". "Phase times" are rooted in the idea that particle positions can be associated with wave packet peaks. Thus following the peak position of a wave packet one could obtain information on the transversal time of the transmitted particle. Consider an initial "minimum" Gaussian packet (having a minimum position-momentum uncertainty product) of average momentum p_0, average position $x_0 < 0$ and spatial variance Δ_x^2,

$$\langle x | \psi(0) \rangle = \frac{1}{[2\pi\Delta_x^2]^{1/4}} e^{-(x-x_0)^2/4\Delta_x^2} e^{ixp_0/\hbar}, \tag{1}$$

colliding with a square barrier of "height" V_0 (in energy units) located between 0 and d. For packets with a well defined momentum (Δ_x large), the point of stationary phase of the integrand of the transmitted packet arrives at $x_1 > d$ at time

$$\tau_{p_0}(x_0, x_1) = m/p_0[(x_1 - x_0) + \hbar\, d\phi_T/dp|_{p=p_0}] \tag{2}$$

where ϕ_T is the phase of the (complex) transmission coefficient, $T_p = |T_p|\, e^{i\phi_T}$. The first term in (2) is nothing but the classical time that a free particle would take to go from x_0 to x_1 with momentum p_0. The second term is the "transmission delay time" and gives the difference between the arrival time at x_1 in the presence of the barrier and the corresponding time in the case of free evolution of the wave packet. From the previous equation it is tempting to interpret

$$\tau_{p_0}^{phase} = m/p_0[d + \hbar\, d\phi_T/dp|_{p=p_0}]. \tag{3}$$

as a "transversal time" across the barrier. The remarkable fact is that, in the tunnelling regime, $\tau_{p_0}^{phase}$ does not increase with the barrier length for sufficiently large barriers. This is the so called "Hartman effect". Thus for large enough barriers anomalously short tunnelling times could in principle be achieved. This is still the case when Dirac's equation is used instead of Schrödinger's equation [9].

The above interpretation rests on the assumption that the packet evolves "freely" out of the barrier region $(0, d)$. However, wave packets with a well defined momentum are necessarily broad in coordinate representation and therefore they are severely deformed both before the hypothetical "entrance" instant $|x_0|\, m/p_0$ and after the "escape" instant $\tau_{p_0}(x_0, x_1) - (m/p_0)(x_1 - d)$, so that they are by no means behaving as free packets before or after the duration of time assigned to the barrier traversal. For arbitrary packets, a common procedure is to define the entrance and escape times by extrapolating the asymptotic free motion of the peaks or centroids of the ingoing and transmitted packets up to their crossing with the barrier edges. But the same type of criticism applies. Actually it is not difficult to find examples where the transmitted packet emerges *before* the incident peak has arrived at the left edge and of course there cannot be causal relation between the two events [10].

The use of wave packets in the tunnelling time conundrum has been critisized because there is no physical law that turns peaks into peaks [2]. However, part of the paradoxical features of wave packet analysis dissapear when one takes seriously the statistical aspect of quantum mechanics, i.e., when one associates an ensemble of systems with the wave packet. The peak is then nothing more than a position of maximum probability and the transmitted packet may be strongly deformed or have several peaks. Moreover, wave packets can be relevant to the analysis of the tunnelling time problem because: (a) Contrary to definitions based on stationary methods the time evolution of the wave packet through the barrier is explicit; (b) Some of the quantities involved, such as the average arrival time [11], see (4) below, are in principle accessible experimentally, and (c) they contain complementary information with respect to the transmission probabilities. This additional information can be useful, for example, to obtain the underlying potential by means of inversion techniques.

Explicit consideration of the statistical nature of the wave packet in the formalism is achieved by averaging the time of arrival using the flux of particles as a distribution of passage times:

$$\langle t \rangle_J (x_1) = \frac{\int_0^\infty J(x_1, t)\, t dt}{\int_0^\infty J(x_1, t)\, dt} \tag{4}$$

The interpretation of this quantity as an arrival time requires the absence of backwards flow (J positive) as well as its essential accordance with the time of absorption by an ideal absorber. A common misconception is that there are not perfect absorbers and that therefore measuring the arrival of particles is necessarily an invasive process where the reflection is unavoidable [12]. But this is not necessarily so: The agreement between measured and calculated collisional cross sections can only be explained by the existence of good enough absorbers. It is indeed possible to build phenomenological optical models which absorb without reflection in a wide momentum range [13, 14, 11]. Assuming perfect absorption, the average time in (4) is arbitrarily close to the average absorption time of the detector [11]. If this is the case, an advantage of this expression is that it is applicable even when a single peak is not clearly defined. In the absence of negative momenta the arrival time at $x_1 > d$ can be expressed as an average of stationary phase times [1]:

$$\langle t \rangle_J (x_1) = \frac{1}{\langle T \rangle} \int_0^\infty dp\, |\langle p | \psi(0) \rangle|^2\, |T_p|^2\, \tau_p(x_0, x_1), \tag{5}$$

where $\langle T \rangle$ is the transmittance.

In order to characterize the tunnelling time a proper definition of the entrance time into the barrier is also required. However, this problem turns out to be more involved since the conditions applicable for the transmitted outgoing flux (J is positive at $x = d$ in an overwhelming fraction of the total time) do not hold at $x = 0$, where there is a mixture of positive and negative fluxes. Selection of the positive part of the flux alone to define the entrance time [15, 16] is not a good option, as discussed in [17–20], because the positive flux cannot be associated with the subensemble of "to be transmitted" particles. Rather it involves both amplitudes, $P\psi$ and $(1-P)\psi$, and actually the "to be reflected" component becomes dominant for wide enough barriers [20]. Since the preparation instant of the initial state is a perfectly known quantity, the best compromise consists in localizing the initial wave packet close enough to the left edge of the barrier with a small spatial variance compared to the barrier length d, in such a way that one can identify the entrance and the preparation instant within a tolerable small uncertainty. However Low and Mende have shown [8] that due to this localization the "usual wave packet analysis", which makes use of the the passage of the peak of undeformed packets to determine the traversal time, fails.

Technically, the failure is due to the incompatibility of a series of conditions:

I. $|x_0| \ll d$. This condition localizes the packet center close to the edge of the potential barrier in a scale determined by d.

II. $\Delta_p = (\hbar/2\Delta_x) \ll p_0$. This is needed by the way in which scattering theory is used in ref. [8] which requires a negligible contribution to the initial wave packet of negative momentum components.

III. The initial wave packet should be confined to the left of the barrier with a negligible penetration through the barrier in comparison with the transmission probability.

IV. In order to follow unambiguously the peak position of the wave packet it must propagate without appreciable deformation with respect to the incident packet. This implies low momenta, $p_0 \ll p_b$, where $p_b = (2mV_0)^{1/2}$ is the barrier height in momentum units.

V. The condition $|T_p\langle p|\psi(0)\rangle|_{p=p_0} \gg |T_p\langle p|\psi(0)\rangle|_{p=p_b}$ is also imposed in ref. [8] to assure that expansions around p_0 can be made.

Low and Mende [8] showed that these conditions are incompatible, and conclude that the usual wave packet formulation necessarily fails for an initial state with x_0 close enough to the barrier to permit an accurate transition time measurement. They speculated that, under conditions I–IV, the transmitted part of the wave packet would be dominated by momentum components with energies exceeding the barrier height. One might think that the localization of the initial wave packet could lead to a large momentum width and consequently to the dominance of large positive momenta over the barrier in the transmitted packet. But since the scale of length used to localize the initial packet is determined by the barrier width such an argument *alone* does not apply for large enough barriers, so the dominance of over the barrier momenta is not obvious. In this work we show that the speculation of Low and Mende is indeed founded.

Nevertheless, conditions I–IV are still too restrictive to guarantee the general validity of the above conclusion. The failure of the formalism under the five conditions mentioned above does not necessarily imply the impossibility of traversal time measurements under less restrictive conditions: I and III are well motivated on physical grounds but conditions II and IV turn out to be too severe. These two last conditions are imposed to assure the applicability of the formalism used, in other words they limit the physical domain where the mathematical treatment used to describe the initial and transmitted packets is valid. But this constraint is excessive. There are physically meaningful *initial* wave packets that cannot be described with the formal treatment in [8], namely, packets with negative momentum components where the standard substitution of scattering theory, see the details in the next section, cannot be made. This indicates that condition II is unnecesary, and that a different mathematical description is required. Actually the arrival time analysis based on Eq. (4) does not require any predetermined form for the arriving wave packet, i.e., it does not require conditions IV and II.

By imposing only the conditions I and III we shall verify by a systematic numerical calculation that for initial states localized close enough to the barrier to permit an accurate transmission time measurement, the process is dominated by over the barrier transmission.

2. Contribution of Negative Momenta

The way to formally handle initial packets with non negligible negative momentum components was discussed in ref. [21]. Here we shall merely review the origin of the difficulties and the main results. Scattering theory allows to identify the amplitudes

$$\langle p | \phi \rangle = \langle p^+ | \psi \rangle \tag{6}$$

where ϕ is the ingoing asymptote of the scattering state ψ, and $|p^+\rangle$ a solution of the Lippmann–Schwinger equation corresponding to an ingoing plane wave of momentum p. Using this property the real packet can be written at any time as a simple integral

$$\langle x | \psi \rangle = \int_{-\infty}^{\infty} dp \langle x | p^+ \rangle \langle p | \phi(0) \rangle \, e^{-iE_p t/h} \tag{7}$$

If the contribution of negative momenta is negligible at $t=0$, [In this work the zero of time corresponds to an instant in which the wave packet is located to the left of the barrier.] $\phi(0)$ can be substituted by $\psi(0)$ in the above expression. However, if negative momenta are present, then the wave packet collides with the barrier at negative times so that it cannot be considered yet the asymptotic incomming free packet. Note that this substitution is central in the treatment made in ref. [8].

Fortunately, the general formula for the transmittance is not affected by the presence of negative momenta [21],

$$\langle T \rangle = \int_0^\infty |T_p|^2 \, |\langle p \,|\, \psi(0) \rangle|^2 \, dp \tag{8}$$

The validity of this formula only requires that the packet does not overlap with the potential barrier at time $t = 0$.

3. Large Barrier Limit of the Hartman Effect

Hartman pointed out, based on a wave packet analysis, the independence of the tunnelling time with respect to the barrier length d in a range of d values. But he also noted that for thick enough barriers, the times become again dependent on d because of the dominance of over the barrier momenta [22]. In the recent literature this fact has been frequently ignored, perhaps because it is not directly evident from the expression (3) which refers to a single plane wave momentum and not to a real packet, but it is central to our arguments. A quantitative characterization of this transition between the two regimes was provided in [1] by giving the equation of the curve separating the two regions in the Δ_x, d plane. The origin of the formula separating the "Hartman plateau" [23] and the regime where over the barrier momenta dominate was however not sufficiently discussed. It is our purpose here to do so.

We first separate the transmittance into tunnelling and over the barrier contributions

$$\langle T \rangle_< = \int_0^{p_b} |T_p|^2 \, |\langle p \,|\, \psi(0) \rangle|^2 \, dp \tag{9}$$

$$\langle T \rangle_> = \int_{p_b}^\infty |T_p|^2 | \, \langle p \,|\, \psi(0) \rangle|^2 \, dp \tag{10}$$

We shall restrict ourselves in what follows to Gaussian packets of the form (1) with $p_0 \leqslant p_b$. Then, in momentum representation

$$\langle p \,|\, \psi(0) \rangle = \frac{1}{[2\pi \Delta_p^2]^{1/4}} \, e^{-(p-p_0)^2/4\,\Delta_p^2} \, \mathrm{e}^{-ix_0 p/\hbar}. \tag{11}$$

Asymptotic expansions of the above two quantities, to leading order, are given by [29]

$$\langle T\rangle_< \sim |T_p|^2_{p=p_0} + O[(\Delta_p)^2] \tag{12}$$

$$\langle T\rangle_> \sim \frac{\Delta_p}{(2\pi)^{1/2}(p_b-p_0)}\, e^{-(p_b-p_0)^2/2(\Delta_p)^2}\, |T_p|^2_{p=p_b} + O[(\Delta_p)^3]. \tag{13}$$

Assuming that we are dealing with an opaque square barrier, (i.e., $\kappa_0\, d \gg 1$, where $\kappa_0=(k_b^2-k_0^2)^{1/2}$, $k_0=p_0/\hbar$ and $k_b=p_b/\hbar$) we have

$$|T_p|^2_{p=p_0} \approx \left(\frac{4k_0\kappa_0}{k_0^2+\kappa_0^2}\, e^{-(\kappa_0 d)}\right)^2 \tag{14}$$

$$|T_p|^2_{p=p_b} \approx \frac{4}{(k_b d)^2} \tag{15}$$

From the above expressions it is clear that for wide enough barriers, the transmittance $\langle T\rangle = \langle T\rangle_< + \langle T\rangle_>$ becomes dominated by the over the barrier contribution. The critical barrier length, d_c, where the transition between tunnelling and over the barrier motion takes place can be obtained by equating Eqs. (12) and (13). Retaining only the dominant exponential dependences one then finds

$$d_c \approx \left(\frac{(p_b-p_0)^3}{p_b+p_0}\right)^{1/2} \left(\frac{\hbar^{1/2}}{2(\Delta_p)}\right)^2. \tag{16}$$

This is the expression given in ref. [1], with p_0 substituted by the momentum of the first resonance. Now it is clear that, on a quantitative setting, by the term "wide enough barriers" (as used above) one really means potential barriers with $d \gg d_c$.

4. Physical Conditions

We would ideally impose only conditions I and III on physical grounds. However, the validity of the formal treatment (in our case asymptotic expansions) requires additional conditions.

Condition III amounts to impose that the initial packet remains essentially in the region on the left of the potential barrier. The criterion is that the the probability to find the particle on the right of $x=0$, $P(x>0,\ t=0)$, should be much smaller than the tunnelling probability for the packet,

$$P(x>0,\ t=0) \ll \langle T\rangle_<. \tag{17}$$

An asymptotic expansion of $P(x>0,\ t=0)$ in terms of the variable $(\Delta_x)/|x_0|$ gives, to leading order [29]

$$P(x>0,\ t=0) = \int_0^\infty |\langle x\,|\,\psi(0)\rangle|^2\, dx \sim \frac{1}{(2\pi)^{1/2}}\, \frac{\Delta_x}{|x_0|}\, e^{-|x_0|^2/2(\Delta_x)^2} + O\left[\frac{(\Delta_x)^3}{|x_0|^3}\right]. \tag{18}$$

Using (17), (11), (14) and (18), and retaining only exponential dependences, the condition III mathematically reads

$$|x_0| \gg \frac{\hbar}{\Delta_p}(\kappa_0 d)^{1/2} \tag{19}$$

which, for $|x_0| \ll d$, basically coincides with Eq. (2.4) of ref. [8].

Using the conditions I and (19), and comparing with (16), we find

$$d \gg \kappa_0(\hbar/\Delta_p)^2 \gg \kappa_0 \frac{p_b - p_0}{p_b + p_0}(\hbar/2\Delta_p)^2 = d_c. \tag{20}$$

In words, by imposing small penetration (with respect to the tunnelling probability) and an initial packet position close to the barrier (with respect to d), the transmission is dominated by momenta over the barrier. Note however that this is not a general result, since the validity of the asymptotic expansions and opaque barrier conditions require

$$|x_0|^{-1} \ll \Delta_p \ll p_0 \ll p_b \tag{21}$$

5. Numerical Results

The previous result, Eq. (20), complements Low and Mende's conclusion because it confirms their speculation about the dominance of momenta over the barrier under the conditions I–IV [The inequalities I, (19) and (21) are essentially the ones considered in ref. [8].] However, one cannot reach this conclusion when negative momenta become important or when the incident energy is not much smaller than the barrier energy, since Eq. (21) is no longer valid. To study these cases, we have numerically computed the ratio

$$r = \frac{\langle T \rangle_<}{\langle T \rangle_>} \tag{22}$$

in the d, p_0 plane for square barriers with different values of p_b that cover three orders of magnitude. The initial packet was chosen as a minimum uncertainty Gaussian with average position $x_0 = -d/10$ and spatial variance Δ_x^2 given implicitly by $P(x > 0, t = 0) = 10^{-4}\langle T \rangle$. The maximum value of r is in all cases ≈ 0.02, see Figs. 1–4. For large barriers, where the Hartman effect can be expected to be of any importance, r becomes negligible. We have also explored other wave packet forms instead of Gaussians, in particular, square packets of the form

$$\langle x | \psi(0) \rangle = \begin{cases} (10/d)^{1/2}\, e^{ip_0 x/\hbar}, & \text{if} \quad d/10 < x < 0 \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

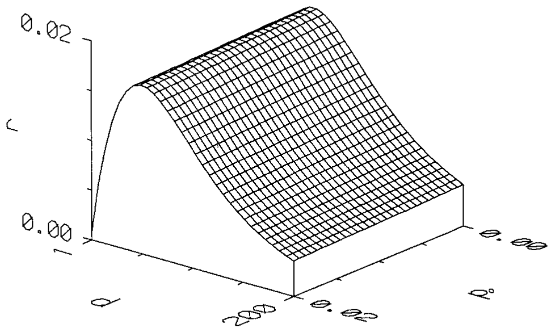

FIG. 1. The ratio $r = \langle T \rangle_{<} / \langle T \rangle_{>}$ as a function of d and p_0 for the wave packet of Eq. (1) with $x_0 = -d/10$ and Δ_x given implicitly by the condition $P(x>0, t=0) = 10^{-4} \langle T \rangle$. The square barrier, of height $p_b = (2mV_0)^{1/2} = 0.02$, is located in the interval $[0, d]$ of the x-axis. All quantities are expressed in atomic units.

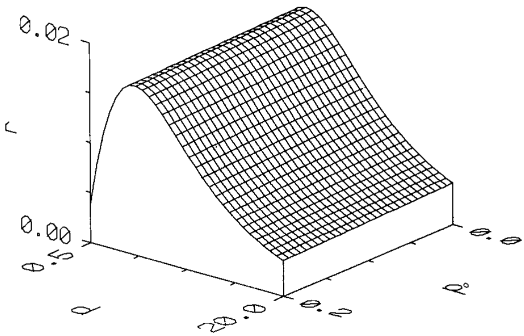

FIG. 2. The same as Fig. 1 with $p_b = 0.2$.

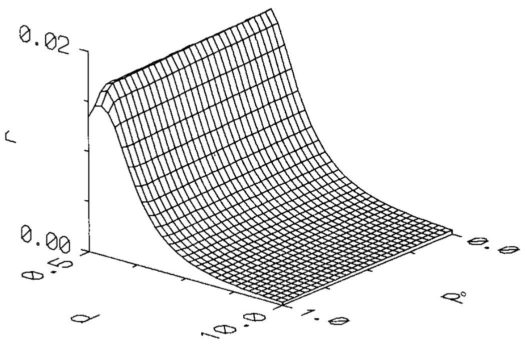

FIG. 3. The same as Fig. 1 with $p_b = 1$.

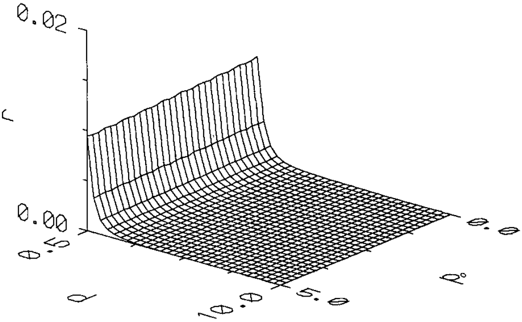

FIG. 4. The same as Fig. 1 with $p_b = 5$.

were used, but the results did not change in any significant manner. In summary, the transmitted part of wave packets satisfying conditions I and III is clearly dominated by momenta over the barrier.

6. Discussion and Conclusions

In this paper we have investigated the possibility of measuring anomalously short tunnelling times by locating the initial packet close to the barrier, and have found a negative conclusion. This work complements previous results by Low and Mende by extending the domain of conditions allowed in the experiment.

There is no single quantity that contains all possible temporal aspects of tunnelling, and different experiments involving different auxiliary variables will require in general different analysis. Many authors have looked at the coupling of the tunnelling motion with other degrees of freedom. Another way is to look directly at the time evolution of the wave packets. In particular, following wave packet *peaks* has been a common approach. When the asymptotically moving peaks are extrapolated to the barrier edges, however, this method gives anomalously short traversal times for these peaks. In fact the "delay time" with respect to the free motion becomes negative and its absolute value increases with the barrier length, in such a way that the tunnelling traversal time becomes independent of the barrier length. This behaviour has been experimentally confirmed for photons [24, 25], and there is no reason to expect anything different in the case of electrons. The question of a possible violation of Einstein's causality immediately comes to mind, since the effect does remain with the wave equation or Dirac's equation. As stated in a recent review: "A simple physical resolution of the appearance of these anomalously short times is still missing" and "this remains an uncomfortable situation" [7]. It is then natural to try to understand and describe this behaviour. An objection to the way in which the seeming short times are found is that the quantum particles should be associated with the packets rather than with the peaks (which, besides, can be multiple) and that the spatial extension of the packet makes in fact the definition of an entrance and of an escape instant from the barrier very uncertain. The concept of "delay" is also misleading if one does not keep in mind that while the free packet peak goes behind the one that traverses the barrier, the total probability to find the particle at the right of a given position can (and in all our numerical computations does) remain larger for the free packet. A possible solution proposed to the paradox of rapid transmission time has been the association of the transmitted peak with the front of the incident wave packet, but this is rather problematic too [26]. There is some support to this idea from the Bohm interpretation of quantum mechanics [6]. However the particle trajectories in this theory depend not only on their initial position but on the full wave, since the quantum potential depends on it. In fact there are other approaches leading to different results for the influence of different regions of the incident packet on the transmitted peak [26]. For relativistic equations, such as the Klein–Gordon equation, the propagation of influences is strictly limited by the speed of light c [27], but this is not the case for the Schrödinger equation. Its Green's function for arbitrarily distant points at arbitrarily small times (other than zero) can be non-zero. Strictly speaking, the full wave packet contributes to the transmittance through the Green's (or influence) function $G(x', t'; x, t)$, see [28] and references therein.

Our approach here is not based on wave packet peaks. Rather, we consider explicitly the statistical nature of wave mechanics and consider a time of arrival averaged over the transmitted flux. Some aspects of the theory required for the analysis have been previously worked out [21, 11]. The initial instant proposed here to be compared with (and substracted from) this arrival time is the initial preparation instant. In order to associate the difference between these two instants with a duration of tunnelling the packet has to be close to the barrier in the scale of the barrier length. This localization also circumvents the problem of associating the transmitted particles with a particular region of the initial packet. However if this condition is enforced, making sure that the packet is well localized to the left of the barrier, no anomalously short average arrival times are found at the right edge, and the transmission is dominated by momenta over the barrier.

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