Dynamical Evolution of a Doubly Quantized Vortex Imprinted in a Bose-Einstein Condensate

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The recent experiment by Shin *et al.* [Phys. Rev. Lett. **93**, 160406 (2004)] on the decay of a doubly quantized vortex is analyzed by numerically solving the Gross-Pitaevskii equation. Our results demonstrate that the vortex decay is mainly a consequence of dynamical instability. The monotonic increase observed in the vortex lifetimes is a consequence of the fact that the measured lifetimes incorporate the time it takes for the initial perturbation to reach the central slice. When considered locally, the splitting occurs approximately at the same time in every condensate.

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Since the creation of the first dilute-gas Bose-Einstein condensates, there has been great interest in characterizing their superfluid properties. This has stimulated a great deal of theoretical and experimental work aimed at studying the rotational properties of dilute Bose gases [1] and, in particular, the nucleation and stability properties of vortices [2]. Numerous experiments have succeeded in generating vortices [3]. In practically all of them, the vorticity appears concentrated in a number of singly quantized vortices. This is a consequence of the fact that multiply quantized vortices are energetically unstable against their splitting in an array of vortices with unit topological charge [4]. Multiply quantized vortices are also dynamically unstable, which implies that they decay even in the zero-temperature limit [5].

Recently, Leanhardt *et al.* [6] obtained multiply quantized vortices in a gaseous Bose-Einstein condensate by using a topological phase-imprinting technique proposed by Nakahara *et al.* [7]. They started from nonrotating ^{23}Na condensates in the $|1, -1\rangle$ and $|2, +2\rangle$ hyperfine states. By adiabatically inverting the magnetic bias field, the initial states were transformed into vortex states with axial angular momentum per particle $2\hbar$ and $-4\hbar$, respectively. This has opened the possibility for studying experimentally the stability of multiply quantized vortices and has stimulated new theoretical work. In particular, Möttönen *et al.* [8] have studied numerically the stability of a doubly quantized vortex in a cylindrical condensate as a function of the (dimensionless) interaction strength per unit length along the condensate axis, an_z, where a is the s-wave scattering length and $n_z = \int |\Psi(\mathbf{r})|^2 dxdy$ is the linear atom density along z. They found a series of quasiperiodic instability regions in the parameter space of an_z [5]. The first two of them (the only ones relevant to this work) correspond to $an_z < 3$ and $11.4 \lesssim an_z \lesssim 16$, respectively. By comparing with the solution of the Gross-Pitaevskii equation for a harmonically trapped cigar-shaped condensate, these authors conclude that a doubly quantized vortex is dynamically unstable, and it is in the above instability regions where the vortex decay is initiated, giving rise in the process to a pair of intertwining singly quantized vortices.

Shortly thereafter, a second experiment was carried out at Massachusetts Institute of Technology (MIT) [9], aimed at studying the characteristic time scale of the splitting process as a function of the interaction strength $an_{z=0}$. Experimental results were obtained from integrated absorption images along the condensate axis after 15 ms of ballistic expansion. A key ingredient was the fact that, in order to increase the visibility of the vortex cores, absorption images were restricted to a 30 μm thick central slice of the condensate. This experiment showed that, even at $T \simeq 0$, doubly quantized vortices decay into two singly quantized vortices on a time scale that is longer at higher atom density, with an observed lifetime that increases monotonically with $an_{z=0}$, showing no quasiperiodic behavior.

These results pose a number of open questions. First, the observed monotonic behavior seems to be not consistent with the local density approach proposed in Ref. [8]. According to this proposal, as the atom density increases, for $an_{z=0} \sim 12$, the second instability region mentioned above arises within the central slice. One then would expect this fact to manifest as a decrease in the observed lifetime, which does not occur. This raises some doubts on the validity of the local density approach [9]. Second, numerical work has appeared recently that suggests that the vortex decay is a consequence of thermal fluctuations [10]. Even though it seems reasonable to attribute the splitting to a dynamical instability, it would be desirable to have theoretical results in good quantitative agreement with the experiment, which could corroborate this assumption. Finally, the detailed dynamical behavior of the vortex along the entire z axis can be especially relevant for characterizing the splitting process in very elongated condensates such as those studied in the MIT experiment. Different regions along the z axis could exhibit different local behavior which could be determinant to understand the process. The experimental results cannot provide this kind of information, and, thus, it would be desirable to complement them with the detailed dynamics of the vortex along the entire condensate.

In this Letter, we address the above questions by performing a realistic computer simulation of the MIT experiment. Our results, which are in very good quantitative agreement with the experiment, enable us to understand the experimental results by providing a detailed visualization of the splitting process. They confirm that the decay of the vortex core is essentially a dissipationless process driven by a dynamical instability. They also indicate that the local density picture proposed in Ref. [8] provides the key ingredients to interpret properly the splitting process. When considered locally, the decay is characterized by the sole parameter an_z, and it is always initiated in those regions along the z axis where $an_z \simeq 1.5$ or $an_z \simeq 13.75$, on a time scale that is in all cases of the order of 15 ms. For large condensates ($an_{z=0} \gtrsim 3$), the lifetime observed in the central slice incorporates the time it takes for the nonlinear perturbation to propagate from the location where the splitting is initiated to the final position where it is eventually detected. This nonlocal process explains the observed monotonic increase in the vortex lifetime and makes the local density picture compatible with the experimental results.

In the zero-temperature limit, the dynamics of dilute Bose gases is accurately described by the Gross-Pitaevskii equation (GPE), which governs the time evolution of the condensate wave function

$$i\hbar \frac{\partial \Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + gN|\Psi(\mathbf{r}, t)|^2 \right)\Psi, \quad (1)$$

where N is the number of atoms, $g = 4\pi\hbar^2 a/m$ is the interaction strength, and $V(\mathbf{r}) = \frac{1}{2}m(\omega_r^2 + \omega_z^2)$ is the external confining potential.

For the ^{23}Na condensates used in the MIT experiment, $a = 2.75$ nm and $\omega_r/2\pi = 220$ Hz. The experimental results were obtained using, without distinction, three different axial trap frequencies $\omega_z/2\pi = 2.7$, 3.7, and 12.1 Hz. Integration of the 3D GPE for such highly elongated condensates is a very demanding computational task. As the system evolves in time, it develops a complex fine structure that can be properly resolved only by using very large basis or gridpoint sets. To verify the convergence and accuracy of our numerical results, we have implemented two different integration methods: a Laguerre-Fourier and a Laguerre-Hermite-Fourier pseudospectral method. The evolution in time has been carried out by a third-order Adams-Bashforth time-marching scheme. The same results have been obtained with the two integration methods and using very different basis sets and time steps.

We have solved the GPE starting, in all cases, from the stationary state compatible with a doubly quantized vortex. A quantum system is dynamically unstable when it is unstable against arbitrarily small perturbations. To determine whether the vortex decay can be unambiguously attributed to a dynamical instability, at $t = 0$, we introduce a small fluctuation in the trapping potential. Specifically, we introduce a 1% quadrupolar perturbation for a short interval of 0.3 ms. Such a small perturbation produces an almost undetectable change in both the energy and the angular momentum per particle ($\Delta E/E$, $\Delta L_z/L_z < 10^{-5}$). In order to compare with the experiment, we have followed the same procedure as used at MIT. Condensates with different values of $an_{z=0}$ were produced by varying N, and integrated absorption images of a 30 μm thick central slice were then generated at different times. In all cases, the initial vortex decayed into a pair of singly quantized vortices. We have also considered the potential effect of dissipative processes by introducing a phenomenological imaginary time, and, for values consistent with the experiment, we have found it to be negligible. The vortex lifetime was inferred from the absorption images by identifying the instant at which two vortex cores can be resolved unambiguously. An example is shown in Fig. 1(a). We have also analyzed the effect of the ballistic expansion by solving numerically the corresponding GPE in a number of representative cases. This process modifies the predicted lifetime by only 1–2 ms approximately [Fig. 1(b)], which is a consequence of the fact that most of the expansion is a mere scale transformation. Since this correction is of the order of measurement uncertainties, we neglect it in what follows.

To investigate the effects of both the strength and the duration of the perturbation, we have considered two additional cases: a 4% and a 1% quadrupolar perturbation acting for 0.3 and 2 ms, respectively. The corresponding numerical results, obtained for an axial trap frequency $\nu_z = 12$ Hz, are shown in Fig. 1(c). Even though the local value of an_z essentially determines where and when the splitting starts, the sole parameter $an_{z=0}$ is not sufficient to characterize completely the observed splitting times in the central slice. For instance, for the 4% quadrupolar perturbation, the predicted splitting time for a condensate with $an_{z=0} = 3.27$ in the trap with $\nu_z = 3.7$ Hz turns out to be 55 ms, whereas for a condensate with the same value of

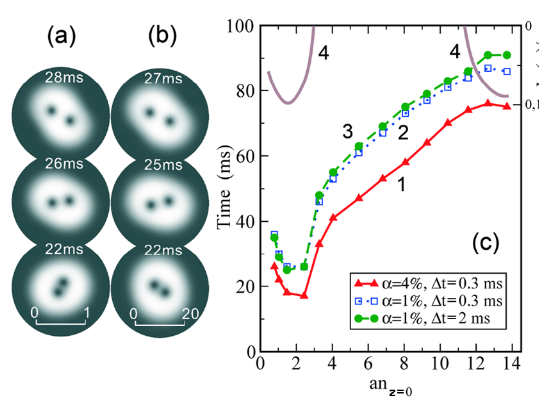

FIG. 1 (color online). (a) In-trap absorption images of a condensate with $an_{z=0} = 1.48$. (b) Same as (a) after 15 ms of ballistic expansion. Lengths are in units of the axial trap $a_z = 6.05$ μm. (c) Predicted splitting times as a function of $an_{z=0}$ for three different perturbations (curves 1–3). Curve 4 shows the excitation spectrum of the unstable modes.

$an_{z=0}$ but in the 12 Hz trap one obtains 33 ms. In general, for $an_{z=0} \gtrsim 3$, the more elongated the condensate, the longer the measured lifetime. This is a consequence of the fact that measured lifetimes incorporate the time it takes for the initial perturbation to reach the central slice. Since the shortest lifetimes occur for the largest axial frequency (12 Hz), this is the only case one has to consider: Experimental data corresponding to two visible cores must lie above the theoretical curve. Note that this does not prevent data corresponding to a single core (those obtained with $\nu_z = 2.7$ or 3.7 Hz) from also lying above the theoretical curve.

While the perturbation strength has an important influence on the predicted lifetimes, its duration seems to be of little importance [Fig. 1(c)]. The main effect of stronger perturbations is to shift the predicted curve toward shorter times. The best agreement with the experiment is found for the 4% quadrupolar perturbation (curve 1). However, the predicted lifetimes are somewhat longer than the experimental ones. Note that the experimental results do not include the 12 ms spent in the inversion of the axial magnetic field B_z (the vortex imprinting). Since the vortex already forms as $B_z = 0$ [11], from this process (the preparation time) one reasonably can expect a contribution of about 5–6 ms. Figure 2 shows the corresponding theoretical curve (from which we have subtracted a conservative amount of 6 ms to account for the preparation time) along with the experimental data of the MIT experiment. The good agreement demonstrates that the decay is driven mainly by a dynamical instability.

During the vortex preparation, as B_z vanishes, all three $F = 1$ components appear in the trap. Thus, for a short interval around the instant of preparation, the spinorial character of the multicomponent Bose-Einstein condensate cannot be neglected [11]. The weak-field seeking state becomes necessarily perturbed by the other components. In the presence of gravitational interaction (perpendicular to the z axis in the experimental setup), each component is shifted from the z axis by a different amount [12]. This introduces an effective nonaxially symmetric perturbation on the dynamical evolution of the weak-field seeking component via the corresponding interaction terms. It seems reasonable to think that the quadrupolar component of this complex effective perturbation might be responsible for the vortex decay.

The theoretical curve in Fig. 2 displays a clear minimum about $an_{z=0} \simeq 1.5$. It is instructive to put this minimum in relation to the excitation spectrum of the unstable (imaginary frequency) modes [see curve 4 in Fig. 1(c)] [5,8]. For small condensates with $an_{z=0} < 1.5$, all of the z axis lies within the first instability region. For such condensates, only a limited subset of the unstable modes associated with that instability region become accessible. The smaller the condensate, the smaller the maximum unstable frequency, and, consequently, the longer the observed decay time. For condensates with $an_{z=0} \geq 1.5$, all of the unstable frequencies become accessible. In such condensates, as the number of particles increases, the first instability region (corresponding to those values along the z axis where $an_z < 3$) moves progressively away from the central slice, towards the edges of the condensate, occupying symmetric positions around $z = 0$. As a consequence, the splitting of the vortex core has to propagate from those regions where it is initiated to the central slice before it can be detected. This nonlocal process is responsible for the monotonic increase in the lifetimes for $an_{z=0} \gtrsim 3$ and explains the minimum predicted about $an_{z=0} \simeq 1.5$. According to a local density picture, in principle, one also would expect a second minimum about $an_{z=0} \simeq 13.75$. Even though no such minimum occurs, a clear change in the slope of the predicted curve can be identify as $an_{z=0}$ enters the second instability region, indicating a different physical behavior.

In Fig. 3, we have calculated the density along the z axis of every condensate (characterized by the different $an_{z=0}$), at the same instant of time ($t = 15$ ms). This axial density has been renormalized in each z plane in such a way that the maximum density in that plane is 1. Thus, for a given $an_{z=0}$, dark zones in the plot density indicate those regions along the condensate axis where the vortex splitting has already begun. As is evident from Fig. 3, when considered as a local process, the splitting takes place approximately

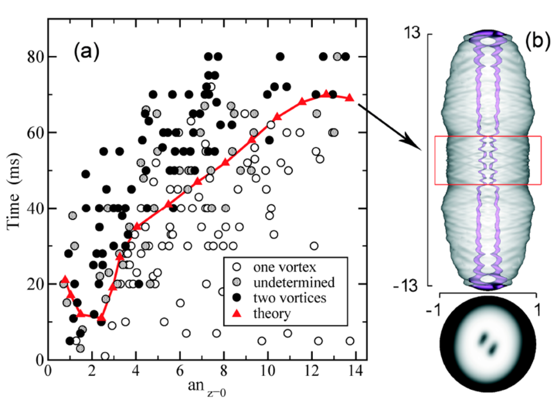

FIG. 2 (color online). (a) Predicted splitting times as a function of $an_{z=0}$ for a 4% quadrupolar perturbation acting during 0.3 ms. (b) Density isosurface of a condensate with $an_{z=0} = 13.75$ at $t = 75$ ms. The corresponding axial absorption image of the central slice (rectangle in the figure) is also shown. Lengths are in units of $a_z = 6.05$ μm.

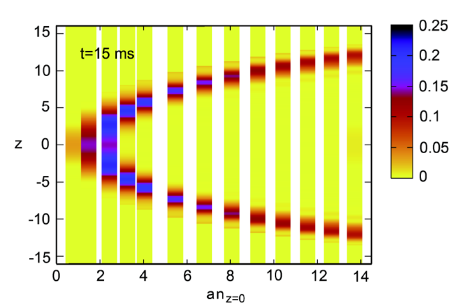

FIG. 3 (color online). Density along the z axis as a function of $an_{z=0}$ at $t = 15$ ms. Lengths are in units of $a_z = 6.05$ μm.

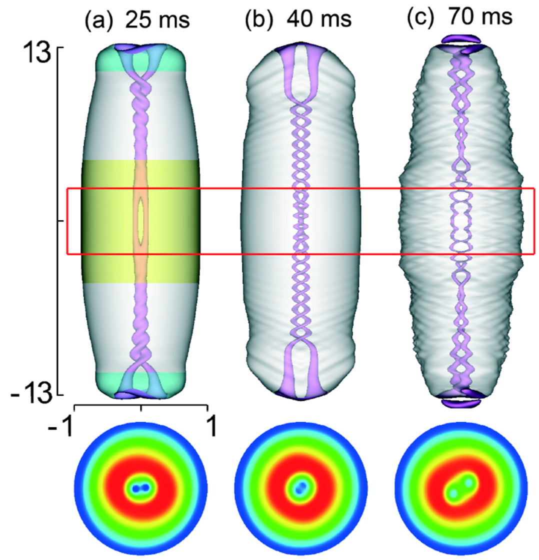

FIG. 4 (color online). Evolution in time of the splitting process of a condensate with $an_{z=0} = 13.75$. The shaded zones in (a) indicate the instability regions. The corresponding axial absorption images of the central slice (rectangle in the figure) are also shown. Lengths are in units of $a_z = 6.05$ μm.

at the same time (on a time scale of the order of 15 ms) in every condensate, regardless of its size. In all cases, the process starts precisely at the predicted instability regions. Figure 3 also shows that for small condensates with $an_{z=0} < 3$ the splitting occurs almost simultaneously along the entire z axis, which is a consequence of the fact that in such condensates the entire z axis lies within the first instability region. As a result, the initial vortex decays into a pair of nearly straight unit-charge vortices. Note that, for $an_{z=0} < 1.5$, the splitting takes somewhat longer times, as expected. As the condensate size increases, the vortex splitting becomes a localized phenomenon that propagates along the z axis, producing in this case a pair of intertwining singly quantized vortices [8], which is a consequence of the dephase in time between different z planes. For $an_{z=0} > 12$, a new instability region appears at the center of the condensate, and the vortex splitting is initiated first at the edges of the condensate and a few milliseconds later at the central slice (a consequence of the smaller value of the corresponding maximum unstable frequency).

Figure 4 shows the evolution in time of the splitting process for a condensate with $an_{z=0} = 13.75$. The first and second instability regions correspond to the shaded zones in Fig. 4(a). This figure clearly shows that, at $t = 25$ ms, the splitting process has already begun in both the edges and the center of the condensate. However, the two vortex cores overlap [Fig. 4(b)] and, thus, cannot be experimentally resolved until much longer times. At $t \approx 70$ ms [Fig. 4(c)], the vortex cores begin to disentangle in such a way that they can be unambiguously resolved at $t = 75$ ms [Fig. 2(b)]. Thus, despite appearances, no contradiction exists with the local density picture. No decrease is observed in the decay times due to the fact that the time required to resolve the two vortex cores is much longer than that it takes for the instability originating at the edges of the condensate to reach the central slice.

In conclusion, we have analyzed the experiment by Shin *et al.* [9] on the decay of a doubly quantized vortex imprinted in ^{23}Na condensates, by solving numerically the 3D Gross-Pitaevskii equation. Our results, which are in very good quantitative agreement with the experiment, demonstrate that the vortex decay is driven mainly by a dynamical instability and allow a better understanding of the splitting process. Despite apparent contradictions, the local density approach is consistent with the experimental results. The monotonic increase observed in the vortex lifetimes is a consequence of the fact that, for large condensates, the measured lifetimes incorporate the time it takes for the initial perturbation to reach the central slice. When considered locally, the splitting occurs approximately at the same time in every condensate, regardless of its size.

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Note added.—After this work was finished, we learned about a very recent preprint [13] in which the same experiment is analyzed.

Note added in proof.—A final version of the above preprint has been published in Ref. [14].

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